

BASICS OF THE COLOR GLASS CONDENSATE *

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I review basic concepts of the effective theory for the color glass condensate which describes the high-energy limit of QCD interactions.

The physics of the color glass condensate [1–3] covers and unifies under the same banner topics which have originally appeared and developed under various names — “small- x physics”, “BFKL evolution”, “unitarity corrections”, “parton saturation”, “multiple pomeron exchanges”, “higher twist effects”, etc. —, but which are modernly understood as manifestations or consequences of the same basic physical mechanism: a change in the form of gluonic matter in the hadron wavefunction at small- x . This change can be visualized as a “critical line” which divides the kinematical plane for deep inelastic scattering into two regions (see Fig. 1): a low density region at high Q^2 (for a given value of x , or $\tau \equiv \ln(1/x)$), in which parton densities evolve according to linear evolution equations (DGLAP or BFKL) and grow rapidly with $1/x$, and a high density regime at relatively low¹ Q^2 , where the parton densities saturate because of the large non-linear effects, and the gluons form a *condensate*. This is a high-density state characterized by an intrinsic scale, the saturation momentum $Q_s(x)$, and by large occupation numbers, of order $1/\alpha_s$, for the gluonic modes with momentum less than or equal to Q_s . The saturation momentum is the typical momentum of the saturated gluons, and grows rapidly with the energy, as a power of $1/x$. The saturation line $Q^2 = Q_s^2(x)$ is the separating line in Fig. 1. Note that the transition across this line is rather smooth, and should not be thought of as a phase transition in the sense of thermodynamics: to my knowledge, no quantity becomes discontinuous at this line. The smooth character of the transition is best demonstrated by the fact that a qualitative

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¹With $Q^2 \gg \Lambda_{QCD}^2$, though: in what follows, I shall always assume weak coupling.

change of behaviour appears already *before* crossing this line, namely, in the region $Q^2 > Q_s^2(x)$, where one sees some more structure in Fig. 1:

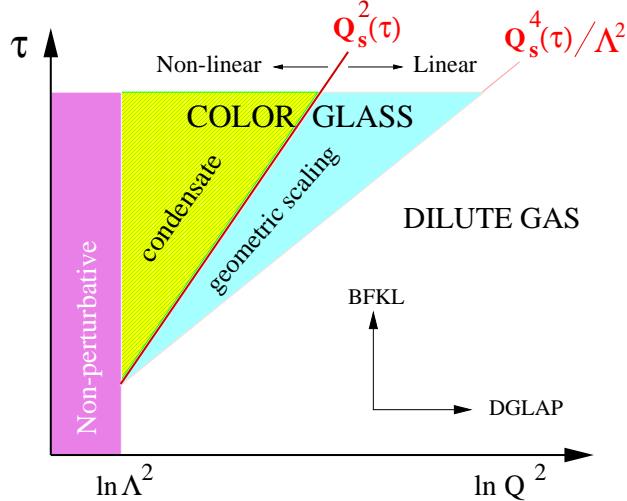


Figure 1: Domains for evolution in the $\tau - \ln Q^2$ plane, with $\tau = \ln(1/x)$.

In addition to the truly dilute regime at very high Q^2 , where the leading twist approximation is excellent and DGLAP equation applies, there is also an intermediate regime in between the saturation line and the “geometric scaling” line [4, 5, 6] where the gluon density is large enough to entail strong correlations. These correlations depend upon Q^2 and x according to *scaling laws* characteristic for a liquid: these are powers of $Q^2/Q_s^2(x)$, with the exponents determined by the BFKL equation. In particular, this behaviour leads to DIS structure functions which violate *strongly* Bjorken scaling (i.e., which show a power-law dependence upon Q^2) [6]. This intermediate region is where BFKL evolution truly applies. Note that, in order to reach higher and higher energies within the BFKL description, one has to simultaneously move up to larger and larger values of Q^2 , in such a way to remain in the linear regime. Thus, strictly speaking, BFKL equation is not the right tool to study the high-energy limit of QCD, defined as the limit $x \rightarrow 0$ at fixed Q^2 . Nevertheless, BFKL provides the right approach towards saturation, and thus determines the scale which plays the decisive role for scattering at very high energies: the saturation momentum.

At all the points on the left of the “geometric scaling” line in Fig. 1 (this includes the BFKL regime and the gluon condensate at saturation), the gluons can be described as a *color glass*. This is a glass because the small- x gluons are typically radiated by color sources (partons) with much larger values of x , and thus larger rapidities, whose internal dynamics is slowed down by Lorentz time dilation. Accordingly, the radiated gluons evolve very slowly relative to natural time scales, so an external probe — like the virtual photon in DIS at small- x — “sees” only a “frozen” configuration of these gluons, which can be any of the configurations allowed by the dynamics of their fast moving sources. Physical observables like cross-sections are then obtained by averaging over all such configurations, with some “weight function” (a functional of the color charge density) which describes the spatial distribution of the sources.

Besides being physically intuitive, this picture is interesting in that it offers a natural framework for the inclusion of non-linear effects like gluon recombination: These effects appear either as non-linearities in the classical Yang–Mills equations which relate the color fields to their sources, or as correlations among these sources, encoded in the weight function. These correlations are built up in the course of the quantum evolution towards small x . This involves a renormalization group analysis [7, 3], in which quantum fluctuations are ‘integrated out’ in layers of rapidity and in the presence of the color fields radiated by the sources constructed in previous steps. This is done in the leading logarithmic approximation with respect to $\ln(1/x)$, to preserve the hierarchy of time scales. The result of this calculation [3] is a functional differential equation which governs the evolution of the weight function with increasing $\tau = \ln 1/x$.

The fact that this is a functional equation means that it is equivalent to an infinite hierarchy of ordinary evolution equations for n -point correlation functions. In particular, if these equations are written for Wilson lines operators (the relevant operators for scattering at high energy), they turn out to be the same as the equations originally derived by Balitsky [8], by using the operator–product expansion near the light-cone. In fact, within the space of Wilson lines, the functional equation for the color glass is equivalent [9] to a similar equation deduced by Weigert [10] from an analysis of Balitsky’s equations.

In the low density regime at $Q > Q_s(x)$, where the color fields are weak, the non-linear effects can be neglected. Then, the equation for the 2-point function reduces to the BFKL equation for the gluon distribution [7, 3]. By

itself, this equation does not describe saturation, but can be used to determine the saturation scale from the condition that the gluon density becomes of order $1/\alpha_s$ when $Q \sim Q_s(x)$. This yields : $Q_s^2(\tau) = Q_0^2 e^{\lambda\tau}$, where only the exponent λ is under control. A leading-order BFKL calculation with fixed coupling constant gives $\lambda = c(\alpha_s N_c / \pi)$ with $N_c = 3$ and $c = 4.88\dots$ [11, 12, 5]. This is of order one for $\alpha_s \sim 0.2$, which is far too large to agree with the phenomenology at HERA within the “saturation model” [14, 4]. Remarkably, however, a recent calculation [13] using the NLO BFKL formalism reduces this value to $\lambda \approx 0.3$, in good agreement with the fits in Refs. [14, 4].

More generally, it has been recently shown [15] that, in the weak field and large- N_c limits, the functional evolution equation for the color glass [3] is equivalent to Mueller’s color dipole picture of the onium wavefunction [16].

But the most interesting regime, of course, is the high-density regime at $Q \leq Q_s(\tau)$. In this regime, the color fields are strong — the typical field strength is of order $1/g$ —, so the non-linear effects must be included exactly. Then, the functional evolution equation is too complicated to be solved exactly (except maybe via numerical simulations), but mean field approximations have been constructed [17], with interesting physical conclusions: First, the gluon phase-space density is explicitly seen to saturate at a value of order $1/\alpha_s$, as expected. Second, the saturated gluons are spatially correlated with each other in such a way to shield their color charges over a transverse area $\sim 1/Q_s^2$. This results in a smoother spectrum at low transverse momenta, which eliminates the infrared problems of perturbation theory. For most purposes, the saturation momentum acts effectively as an infrared cutoff. This justifies a posteriori the use of perturbation theory. Third, when applied to deep inelastic scattering (and also to high-energy onium–onium scattering [16, 15]), saturation ensures the unitarization of the S -matrix at fixed impact parameter [6].

These conclusions are corroborated by various, analytic and numerical, investigations of the Kovchegov equation [18], which is a simple (since closed) non-linear evolution equation for the scattering amplitude, and may be seen as a special approximation to the first equation in the hierarchy by Balitsky. This equation is a convenient laboratory to study conceptual and phenomenological consequences of saturation, with many recent applications. I refer to [1] for a recent review and more references, and also to several contributions to this meeting which have addressed, more or less directly, this topic [19].

To summarize, the color glass effective theory provides an unified description of the BFKL regime, of the physics of saturation, and of the transition between these two regimes. It also applies [2] to large nuclei ($A \gg 1$), which involve high gluon densities even at not so small values of x (say, $x \sim 10^{-2}$, as in the experiences at RHIC), because of the many tree-level color sources: the $3A$ valence quarks. There have been many interesting applications of this formalism to the phenomenology of heavy ion collisions at RHIC, that I have no time to discuss here, but for which I refer to the recent review paper [1].

References

- [1] For a recent review, see E. Iancu and R. Venugopalan, *The Color Glass Condensate and High Energy Scattering in QCD*, hep-ph/0303204.
- [2] L. McLerran and R. Venugopalan, *Phys. Rev.* **D49** (1994) 2233; *ibid.* **49** (1994) 3352; *ibid.* **50** (1994) 2225.
- [3] E. Iancu, A. Leonidov, and L. McLerran, *Nucl. Phys.* **A692** (2001) 583; *Phys. Lett.* **B510** (2001) 133; E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, *Nucl. Phys.* **A703** (2002) 489.
- [4] A. Staśto, K. Golec-Biernat, J. Kwieciński, *Phys. Rev. Lett.* **86** (2001) 596.
- [5] E. Iancu, K. Itakura, and L. McLerran, *Nucl. Phys.* **A708** (2002) 327; A. H. Mueller and D.N. Triantafyllopoulos, *Nucl. Phys.* **B640** (2002) 331.
- [6] For more details see K. Itakura, contribution to DIS03, these proceedings.
- [7] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *Nucl. Phys.* **B504** (1997) 415; *Phys. Rev.* **D59** (1999) 014014.
- [8] I. Balitsky, *Nucl. Phys.* **B463** (1996) 99; hep-ph/0101042.
- [9] J. P. Blaizot, E. Iancu and H. Weigert, *Nucl. Phys.* **A713** (2003) 441.
- [10] H. Weigert, *Nucl. Phys.* **A703** (2002) 823.
- [11] L.V. Gribov, E.M. Levin, and M.G. Ryskin, *Phys. Rept.* **100** (1983) 1.
- [12] A. H. Mueller, *Nucl. Phys.* **B558** (1999) 285.

- [13] D.N. Triantafyllopoulos, *Nucl. Phys.* **B648** (2002) 293.
- [14] K. Golec-Biernat and M. Wüsthoff, *Phys. Rev.* **D59** (1999) 014017.
- [15] E. Iancu and A. H. Mueller, hep-ph/0308315.
- [16] A. H. Mueller, *Nucl. Phys.* **B415** (1994) 373; *ibid.* **B437** (1995) 107.
- [17] E. Iancu and L. McLerran, *Phys. Lett.* **B510** (2001) 145;
E. Iancu, K. Itakura and L. McLerran, *Nucl. Phys.* **A724** (2003) 181.
- [18] Yu. V. Kovchegov, *Phys. Rev.* **D60** (1999) 034008; **D61** (2000) 074018.
- [19] J. Bartels, H. Kowalski, M. Kozlov, E. Levin, M. Lublinsky, A.H. Mueller,
S. Munier, M.G. Ryskin, A. Stašto, these proceedings.